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POSSIBILITY OF SURVEYING ITALIAN EARTHQUAKES
BY MEANS OF A NETWORK OF STRONG MOTION ACCELEROGRAPHS

E. Iaccarino

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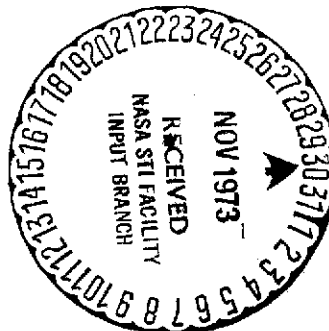
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16. Abstract After having examined the necessity of installing in Italy a network of strong motion accelerographs, a study was carried out to evaluate the probability of picking up by means of this net- work an event of intensity VI or more. In addition, the number of accelerograms obtained each year was calculated. This study was performed by using the results of the statistical analysis of the areas surrounded by the isoseismals of 470 earthquakes experienced in Italy from 1893 to 1965. Moreover, for a regular mesh network, it was possible to examine how the results vary with the distance between two instruments located at the tie point of the mesh. At least, since these results are theoretical, it has been shown how to evaluate actual cases for a network of any shape.					
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POSSIBILITY OF SURVEYING ITALIAN EARTHQUAKES
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E. Iaccarino
CNEN, Safety and Control Division

1) Introduction

/1*

The fundamental problem of seismic engineering is that of obtaining starting data for the designing of structures. These data are as follows: a) the maximum acceleration of the seism; b) the response spectrum (accelerogram).

Strong motion accelerographs record the acceleration in velocity of a seism and seismoscopes record the maximum amplitude of a simple oscillator subjected to a seism; but this they do only for the spot where they are installed. Three seismoscopes of different periods installed at the same place would suffice for evaluation of the ground's response spectrum. Up to the present time only a few accelerographs have been installed in Italy; therefore, inasmuch as strong seisms are rare events, not many have been recorded.

More abundant, on the other hand, is the information about earthquakes obtained by means of macroseismic observations or seismographic recordings. All known Italian earthquakes have been catalogued and classified according to the Mercalli intensity scale (see Ref. 1). But this classification is purely subjective and has no scientific foundation. It is based chiefly on evaluation of damage to buildings and is therefore subject to different interpretations from zone to zone and cannot take into account the evolution of building construction techniques over the centuries.

*Numbers in the margin indicate pagination in the foreign text.

What data are available have been utilized to develop some empirical correlations among intensity, magnitude and acceleration. Of these, the relation of Cancani Sieberg is the one that has up to now permitted evaluation of the acceleration in ground motion by means of macroseismic intensity. Unfortunately, the seismographic stations that went into operation at the turn of the century and that permitted the above-mentioned correlation to be developed, utilized apparatus whose measurements do not appear very reliable in this day and age. It has only been in the last decade that some stations have put into operation more perfected seismographs. But it must be borne in mind that seismographic stations are designed and put into operation by geophysicists, who are principally interested in studying the earth (localization of epicenters and hypocenters, study of the discontinuity of the earth's crust, etc.) and do not take into account the problems of seismic engineering. The fact is that the instruments utilized in seismographic stations are not suitable for high-intensity earthquakes because they jump the scale if subjected to rather violent shocks. Furthermore, the geophysicist, concerned as he is that local ground conditions should not appreciably modify the seismogram, tends to install his apparatus on rocky ground of such a nature that what is picked up is chiefly earthquakes coming from far away. Seismic engineering, on the other hand, cannot leave out of consideration local ground conditions or whatever else may influence the value of the local acceleration produced by the seism. The correlation of Cancani Sieberg therefore appears to be of little use for engineering problems; nor is it convenient to modify it with the data furnished by modern stations that have recently gone into operation.

On the other hand, the situation in the United States is different. In California an extensive network of strong motion accelerographs designed especially for engineering purposes has

been in operation for some time. Therefore it was possible not only to formulate more reliable correlations between intensity and acceleration, especially for high-intensity earthquakes; but studies in depth were also developed to evaluate the local influence of the ground. It turns out that the result of statistical analysis of California earthquakes has shown that the maximum acceleration corresponding to a given intensity, so far from being unequivocal, varies sharply from place to place. Furthermore, it is not even possible to say of two earthquakes of the same nature -- for example, the same magnitude and the same intensity, the same hypocentral depth -- that the distribution of the accelerations in the affected zones is the same. Hence these studies have disclosed all the parameters that, for a given earthquake intensity, influence the local value of the acceleration. Not only this; from the accelerograms available for actual cases it has even been possible to formulate criteria for the evaluation of the response spectra of a determinate site. /3

It is clear that the results of the California studies cannot be extended a priori to other parts of the globe and so not to Italy, either. In Italy, however, such studies are often resorted to for engineering purposes, especially if one is obliged to carry out precautionary evaluations of the design parameters of a structure whose collapse might entail rather serious risks (for example, for nuclear plants).

It follows from all these considerations that in Italy, too, it is necessary to install a network of strong motion accelerographs, designed especially for engineering purposes and in such a manner as to furnish a sufficient number of accelerograms in the shortest time.

The purpose of the present study is to indicate the basic data and the methodologies required to calculate what will be, in

a determinate period of time, the order of magnitude of the number of accelerograms that can be obtained with a network of accelerographs installed in a country like Italy.

The study was divided into three parts:

a) In the first part, historical earthquakes of a given epicentral intensity are analyzed and a statistical analysis of the areas surrounded by the isoseismals of these earthquakes is 74 carried out;

b) In the second part, we present theoretical considerations that make it possible to evaluate the probability of picking up a seismic event by means of a regular mesh network of accelerographs, and the number of accelerograms that can be obtained each year;

c) In the third part, finally, we present some practical considerations about the installation of a network of accelerographs on Italian soil.

2) Results of the Statistical Analysis of Areas

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The statistical analysis of areas (see Appendix) was carried out on 470 earthquakes of intensity VI or more that were experienced in Italy in the period between 1893 and 1965. For these earthquakes it was possible to trace out the isoseismals and to evaluate the areas delimited by each isoseismal in the zones struck by the earthquake.

This analysis concerns the quest for the statistical law of distribution of these areas that will furnish the necessary data for evaluating the probability that an earthquake of degree j at its epicenter will be felt in a determinate area as being at least of degree i , which is that of the isoseismal that surrounds it. For this reason, the areas were subdivided into series of homogeneous values A_{ij} , for each one of which the analysis was carried out.

It was assumed that the lognormal distribution is the law that best fits each series of values. Using lognormal probabilistic paper, therefore, it was possible to carry out a linear interpolation for each series by means of the least squares method.

Figure 1 of the Appendix shows the results of interpolation of the data relating to areas of degree VI, corresponding to an epicentral intensity of degree VI. This evaluation was carried out with the data for 253 earthquakes.

For the sake of convenience, in carrying out our calculations we adopted a subscript t that indicates progressively the order of the isoseismals that succeed one another in an earthquake, starting from the epicentral isoseismal toward the most external one. In particular, with $t = 0$ are indicated all the series formed by the values of the first areas of earthquakes corresponding to intermediate intensity (VI-VII, VII-VIII, VIII-IX, etc.); with $t = 1$, all the series formed by the values of the first areas of whole intensity (VI, VII, VIII, etc.); with $t = 2$, all the series formed by the values of the second areas; with $t = 3$, those formed by the values of the third areas, etc. /6

It should be specified that by the first area is meant the area of the zone in which the seism is felt with the same degree of epicentral intensity while by the second and third, etc., areas are meant successively the areas included by the second, third, etc. isoseismal.

The calculation results of the statistical analysis enabled us to formulate the following hypotheses, checked by means of analysis of variance:

1) It is possible to assume the same lognormal distribution for each series of values. This means that the root-mean-square deviation of each series is the same, or that on probabilistic paper, the lines representing the cumulative probability function have the same slope.

2) The mean value of the areas is the same for all the series corresponding to a subscript t . That is to say, the extension of the area of a seism that presents itself with a determinate probability is independent of the epicentral intensity but does depend on the position that it occupies with respect to the epicentral intensity.

3) The ratio between the mean values of the first areas of seisms of whole and intermediate intensities (with subscripts $t = 1$ and $t = 2$) is equal to 2.291.

4) The ratio between the mean values of two series of consecutive areas (i.e., with subscripts $t + 1$ and t) is always equal to 2.291.

Denoting by R_{ij} the radius of the circle equivalent to area A_{ij} , it turns out that the probability that an area of radius $\leq R$ will be found where a seism of degree j will be felt to be at least of degree i will be: /7

$$P_i \left\{ R \right\} = 1,49343 \int_0^R e^{-\frac{(\log R - 0.64933 - 0.18(t-1))^2}{0.1428}} \frac{dR}{R} \quad (1)$$

where the subscripts i , j and t are connected by the considerations set forth above.

Inasmuch as the cumulative probability curves are parallel, the series with subscript t and those with subscript $t + z$ are connected by the relation:

$$P_{t+z} \left\{ \log R + 0.18z \right\} = P_t \left\{ \log R \right\} \quad (2)$$

Our hypotheses do not, however, permit us to say that ratio A_{t+1}/A_t is statistically distributed about a mean value, but that it is possible, in order to facilitate calculations, to use a single probability curve (see Fig. 4 in the Appendix) and to pass from one series to the other by means of relations (1) and (2).

3. Probability of Picking up a Seismic Event and Number of Accelerograms Obtainable Each Year /9

The probability of picking up an earthquake as a shock of a determinate degree by means of an apparatus sensitive to such a shock, will be given by the product of the probability that the epicenter will be located at such a distance from the instrument as to be picked up, by the probability that the event will take place.

To proceed to this calculation, we made the following hypotheses:

- a) On the ground where earthquakes can happen is installed a network of accelerographs arranged at the vertices of a regular lattice with square meshes of side λ ;
- b) The accelerographs are adjusted for a threshold acceleration value equal to 0.01 g and this acceleration corresponds to a seism of degree VI;
- c) The epicenters can be located indifferently at any point of the ground where the devices are installed;
- d) In the affected zone, the results of the statistics of the areas discussed in the preceding section are applicable.

Let us consider a mesh of the network with side λ , at the vertices of which accelerometers S are placed. In order for an

earthquake affecting an area of radius R to be picked up by these devices, its epicenter will have to fall within the dashed zone $a_1 + a_2$ of Fig. 1.

Generally, if p_r is the probability of picking up an event with such a network, we will have:

$$p_r = (a_1 + a_2)/l^2 \quad (3)$$

and in particular:

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$$p_r = \frac{\pi R^2}{l^2} \quad \text{for} \quad R \leq \frac{l}{2} \quad (4)$$

$$p_r = \frac{\pi R^2}{l^2} \cos \alpha + \frac{\pi R^2}{l^2} \left(1 - \frac{4\alpha}{\pi}\right) \quad \text{for} \quad \frac{l}{2} \leq R \leq l \sqrt{2}/2 \quad (5)$$

$$p_r = 1 \quad \text{for} \quad R \geq \frac{l \sqrt{2}}{2} \quad (6)$$

In (5) we have:

$$\cos \alpha = \frac{l}{2R} \quad (7)$$

Figure II shows the diagram of p_r as a function of ratio R/l . The probability of picking up a seism of degree j at least as a shock of degree i will be given by the product of the probability p_r of picking up the event by the probability p_t that the event will take place.

It follows from the preceding section that:

$$p_t = \frac{dP_t}{dR} \quad (8)$$

where subscript t corresponds to the area of degree i of the seism of degree j at the epicenter.

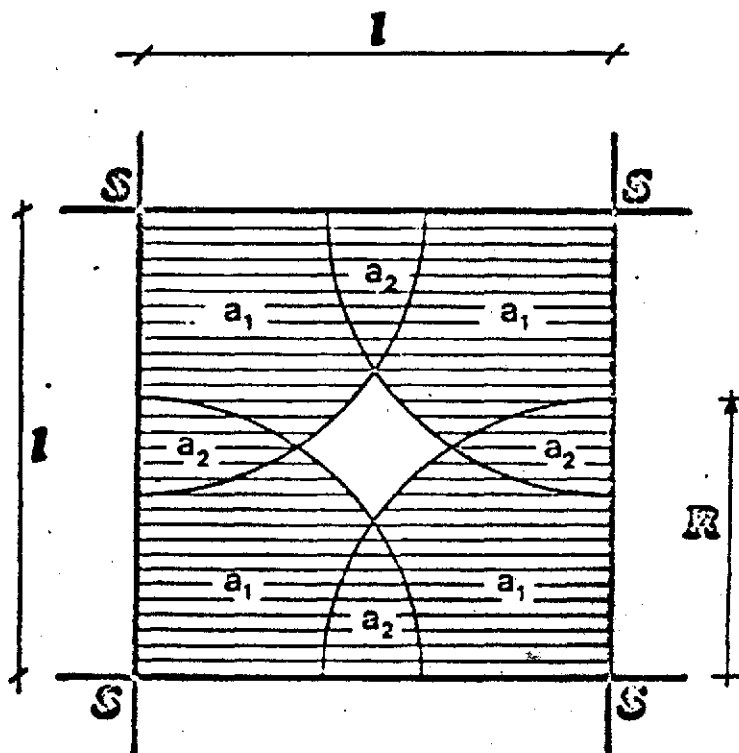


Fig. I.

the above-mentioned 470 earthquakes that were experienced as degree j:

$$F_j = \nu_j / 470 \quad (10)$$

The probability of picking up the 470 earthquakes under consideration at least as shocks of degree i will be:

$$P_i = \sum_{j=1}^{13} P_{jt} F_j \quad (11)$$

The integral extended to the entire range of R:

$$P_{it} = \int_0^{\infty} P_{jt} P_t dR \quad (9)$$

represents the probability of picking up all the seisms of degree j as a shock of degree i for a given value of t.

Now let F_j be the frequency of seisms of degree j in the 470 earthquakes that were considered in the statistical analysis of areas, viz., the quota part of

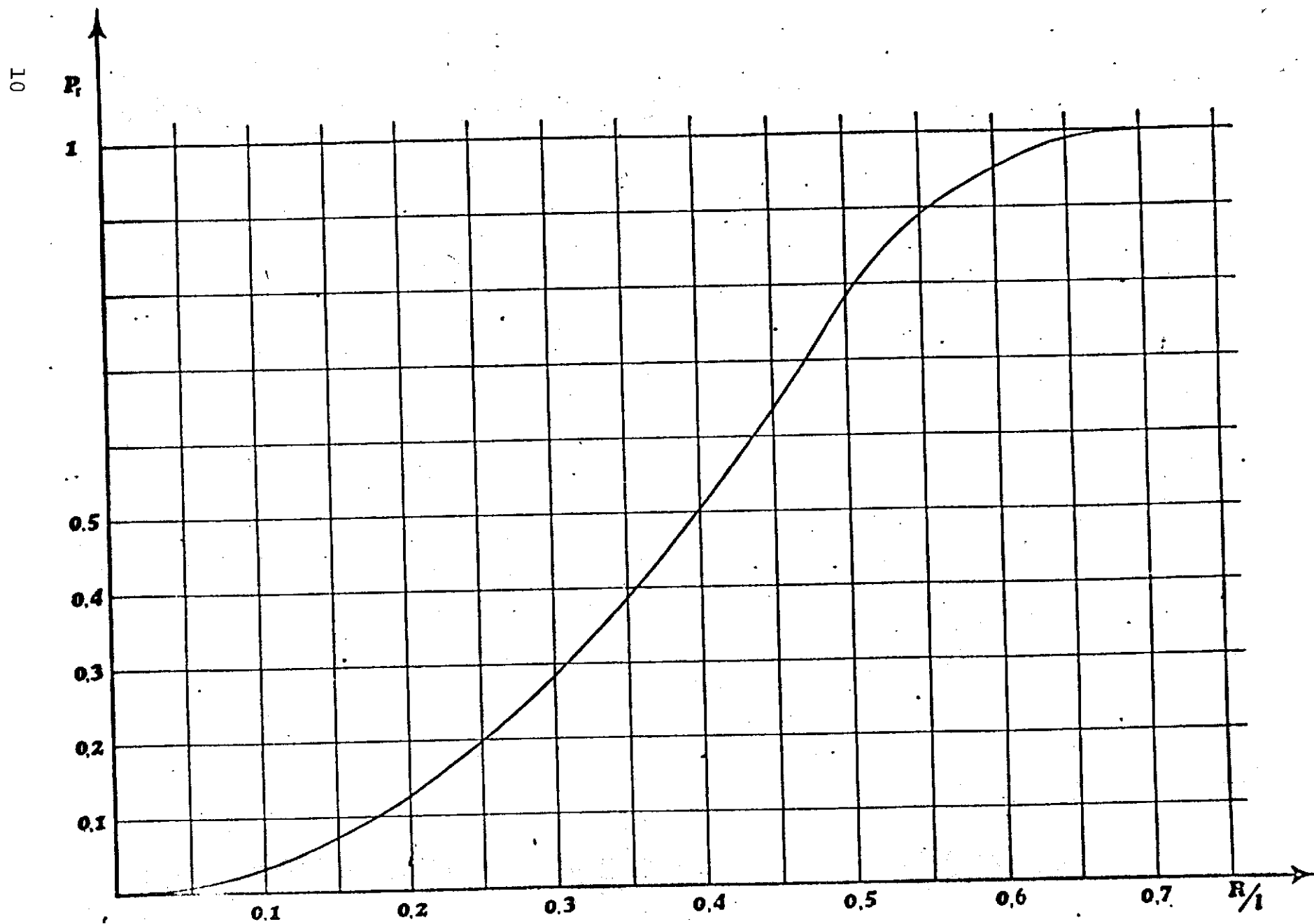


Fig. II.

with i going from 1 to 13 as defined by the convention in the Appendix and P_{rt} , which depends on the pair of subscripts i and j under consideration.

The probability of picking up the 470 earthquakes under consideration by means of devices that can register at least accelerations of 0.01 g will therefore be:

$$P_v = \sum_{j=1}^{13} P_{rt} F_j \quad (12)$$

for j going from 1 to 13.

Multiplying this probability by the annual frequency of earthquakes M , we will get the number of seisms $M P_{VI}$ that it will be possible to pick up each year by means of an accelerograph network with square meshes of side λ .

This calculation method was adopted for a network with 12 regular meshes of side λ , which was made to vary from 0 to 100 km.

Table I shows for different values of λ the probabilities P_{rt} , $P_{rt} F_j$, P_{IV} and the number $M P_{VI}$ of earthquakes that can be picked up each year. It can be inferred from this table that extrapolation of the statistical analysis to areas of high-intensity earthquakes does not call into question the results in spite of the imprecision of the values of F_j that correspond to them. As a matter of fact, in view of the great extension of the area of degree VI that is associated with them, the probability P_{rt} of picking them up is about 1 and the product $P_{rt} F_j$ does not alter the result very much.

As a function of side λ of the mesh we plotted the values of P_{rt} (Fig. III) and the products $M P_{VI}$ (Fig. IV)¹, where

¹[Translator's note: Figure IV is missing.]

j	v_j	F_j %	TABLE I. P_{rt}												
			t \ l	5	10	15	20	25	30	35	40	45	50	75	100
			0	0.745	0.392	0.219	0.134	0.089	0.063	0.047	0.036	0.029	0.023	0.010	0.006
1	253	53.8298	1	0.894	0.608	0.396	0.266	0.187	0.137	0.103	0.081	0.064	0.052	0.024	0.013
2	46	9.7872	2	0.968	0.802	0.613	0.461	0.350	0.270	0.212	0.170	0.139	0.115	0.053	0.030
3	107	22.7660	3	0.993	0.926	0.806	0.677	0.562	0.465	0.387	0.324	0.274	0.233	0.117	0.068
4	24	5.1064	4	0.998	0.980	0.927	0.852	0.766	0.682	0.603	0.532	0.470	0.416	0.236	0.146
5	15	3.1915	5	0.999	0.996	0.980	0.950	0.906	0.854	0.799	0.742	0.686	0.633	0.421	0.286
6	3	0.6383	6	0.999	0.999	0.996	0.987	0.972	0.951	0.924	0.892	0.857	0.821	0.638	0.486
7	14	2.9787	7	1	0.999	0.999	0.997	0.994	0.988	0.979	0.967	0.952	0.935	0.824	0.701
8	4	0.8511	8	1	1	1	0.999	0.998	0.997	0.995	0.992	0.988	0.983	0.937	0.867
9	1	0.2128	9	1	1	1	1	0.999	0.999	0.999	0.998	0.997	0.996	0.983	0.956
10	1	0.2128	10	1	1	1	1	1	1	1	0.999	0.999	0.999	0.994	0.987
11	1	0.2128	11	1	1	1	1	1	1	1	1	1	1	0.999	0.999
12	0	0	12	1	1	1	1	1	1	1	1	1	1	1	1
13	1	0.2128	13	1	1	1	1	1	1	1	1	1	1	1	1
470	100														

1	$P_{rt}F_j$											
	5	10	15	20	25	30	35	40	45	50	75	100
	48.124	32.729	21.317	14.319	10.066	7.375	5.544	4.360	3.445	2.799	1.292	0.700
	9.474	7.849	6.000	4.512	3.426	2.643	2.075	1.664	1.360	1.126	0.519	0.294
	22.607	21.081	18.349	15.413	12.794	10.586	8.810	7.376	6.238	5.304	2.664	1.548
	5.096	5.004	4.734	4.351	3.911	3.483	3.079	2.717	2.400	2.124	1.205	0.746
	3.188	3.179	3.128	3.032	2.891	2.726	2.550	2.368	2.189	2.020	1.344	0.913
	0.638	0.638	0.636	0.630	0.620	0.607	0.590	0.569	0.547	0.524	0.407	0.310
	2.979	2.976	2.976	2.970	2.961	2.943	2.916	2.880	2.836	2.785	2.454	2.088
	0.851	0.851	0.851	0.850	0.849	0.849	0.847	0.844	0.841	0.837	0.797	0.738
	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.212	0.212	0.212	0.209	0.203
	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.211	0.210
	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213
	0	0	0	0	0	0	0	0	0	0	0	0
	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213	0.213
$P_{vi} =$	93.809	75.159	58.843	46.929	38.370	32.064	27.263	23.629	20.707	18.370	11.528	8.176
12 $MP_{vi} =$	7.73	6.20	4.85	3.87	3.16	2.64	2.25	1.95	1.71	1.51	0.95	0.67

$M = 8.24286$, as appears from the report of Ref. 1.

We can infer from Fig. IV that an accelerograph network having a side of 30 km would permit the picking up of only 2.6 earthquakes each year.

When area A_{ij} reaches an extension for which R_{ij} is greater than $l/2$, it is probable that the seism will be picked up by more than one device; for this reason the number of accelerograms that can be obtained each year will be greater than the number of earthquakes that can be picked up.

It can be seen from Fig. I that the a_1 and a_2 are areas for which, if the epicenter of an earthquake of radius R should fall within them, the event would be picked up once or twice, respectively.

In general, denoting by a_k that part of the area l^2 for which, if the epicenter affecting an area of radius R should fall within it, the seism would be picked up k times, the probability of picking up that earthquake k times will be a_k/l^2 .

The number of accelerograms that can be obtained will be given by the summation of the products of these probabilities by the number of accelerograms for each area a_k .

[Note: Page 13 is missing from original.]

with j going from degree i to XII; it will be recalled that in [17] subscript t corresponds to the pair of subscripts i and j under consideration. /14

For a value of i corresponding to degree VI we will have the number of accelerograms obtainable each year with a network of side l .

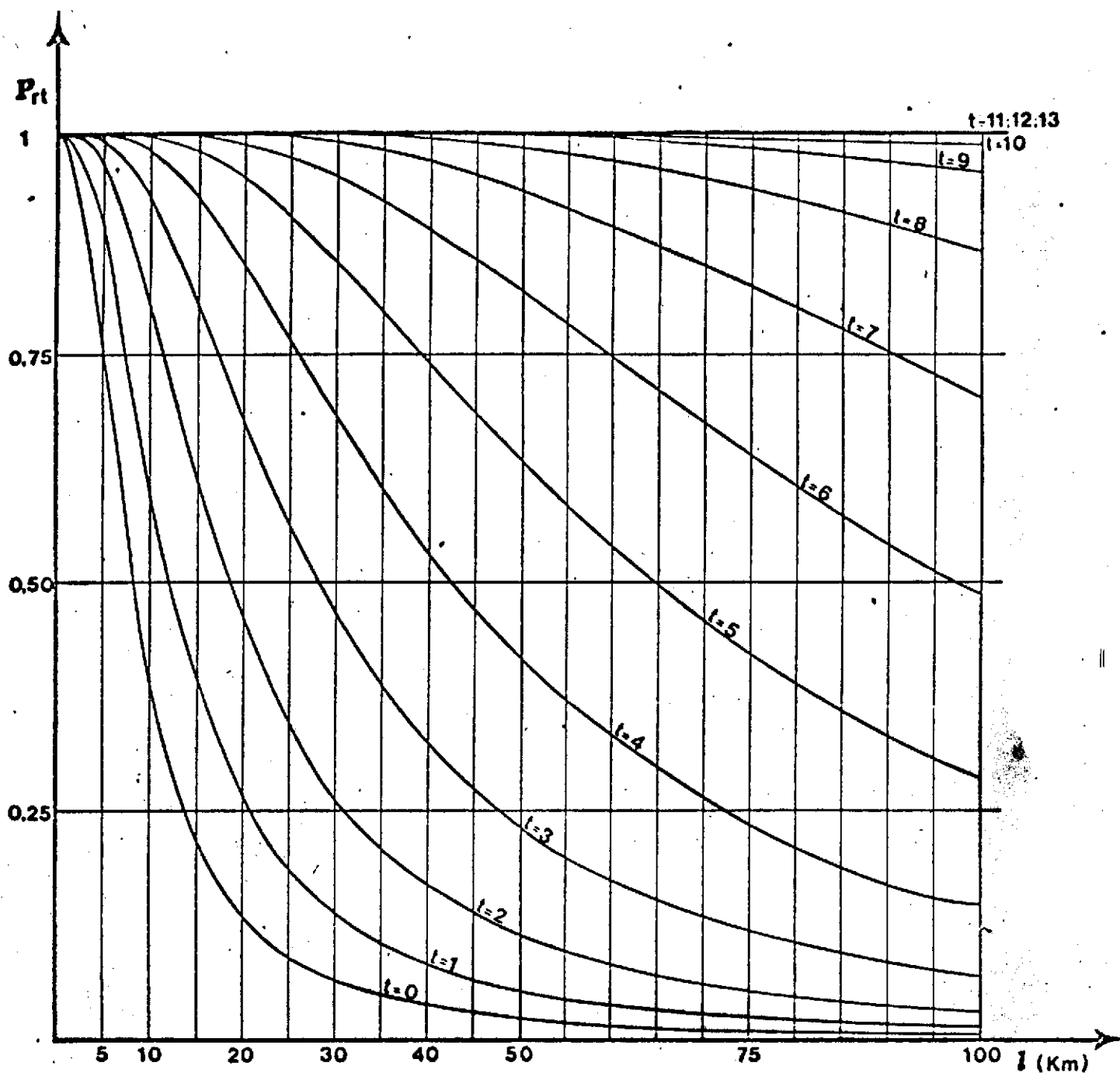


Fig. III.

The number of recordings n_i refers to the total number of recordings obtainable as degree i and more inasmuch as the areas under consideration are those internal to each isoseismal. The number of effective accelerograms of degree i will therefore be:

$$n_i^* = n_i - n_{i+1}$$

(18)

This method is not applicable to seisms of a high degree for which the period of years of observation is not enough to determine the effective values of F_j ; extrapolation of the results of the statistical analysis to the areas would in fact lead us to consider mean areas much greater than those that were actually involved on the occasion of the few events that were experienced in the period under consideration.

For these reasons the method we have been considering was not applied to earthquakes above degree VIII-IX. For the latter, the number of recordings was obtained by simply dividing the values of the areas affected by the earthquakes by λ^2 (the area of the network mesh) and by 70 (the period of years under consideration).

The results of all the calculations for different values of λ are shown in Table II.

Figure V is a semilogarithmic diagram showing the total number of recordings as a function of side λ of the mesh.

It can be seen from this figure that with an accelerograph network with a side of 30 km we can expect to have an average of 7.4 recordings per year.

4. Installation of an Accelerograph Network on Italian Soil /15

The results obtained in the preceding sections are based on the theoretical hypothesis of uniform distribution of accelerographs on the territory affected by the seisms. But the devices require maintenance and periodic check-ups, and it would therefore

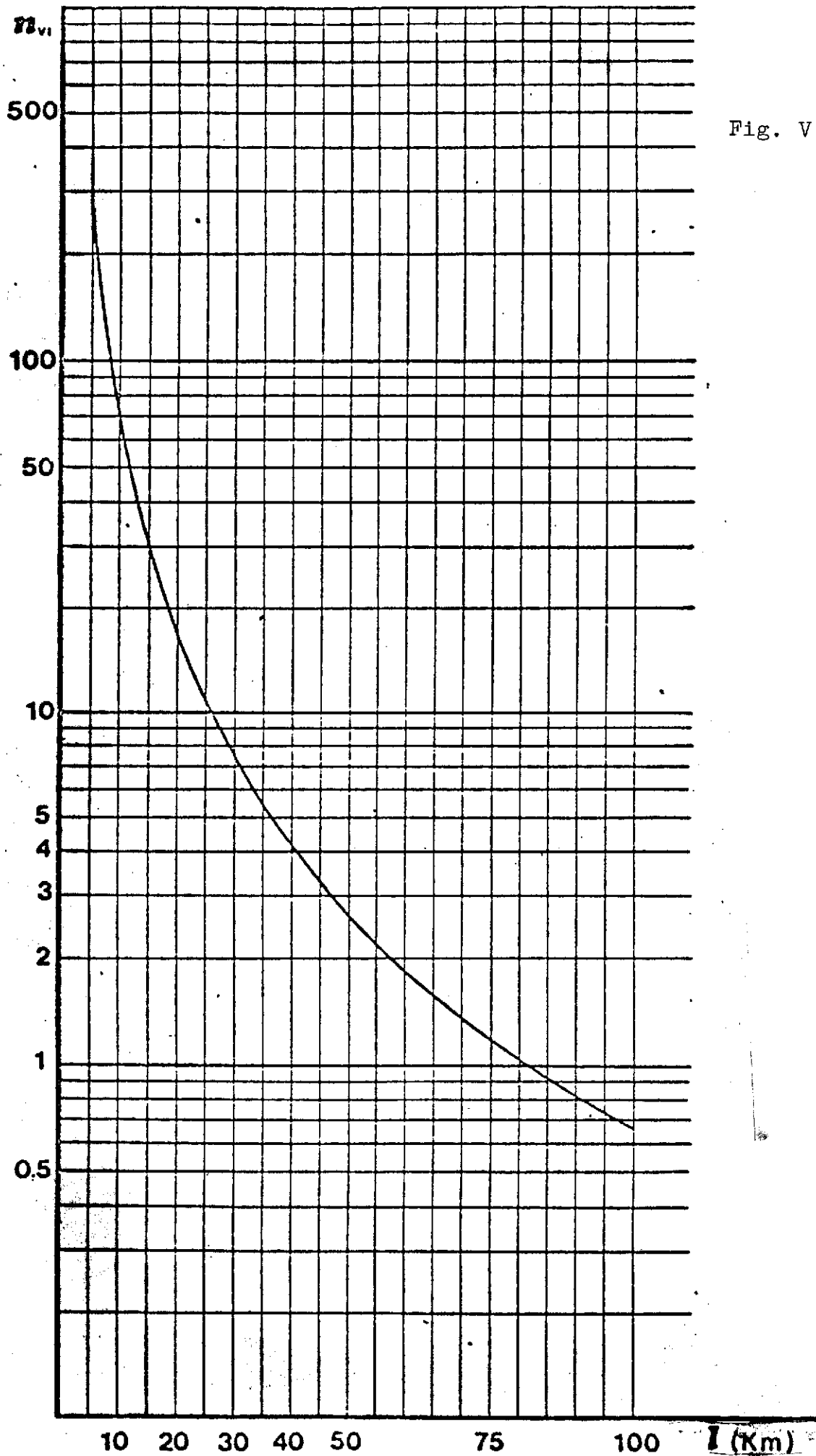


TABLE II.

		I											
	t	5	10	15	20	25	30	35	40	45	50	75	100
	0	2,324	0,581	0,258	0,145	0,093	0,065	0,047	0,036	0,029	0,023	0,010	0,006
	1	5,325	1,331	0,592	0,333	0,213	0,148	0,109	0,083	0,066	0,053	0,024	0,013
	2	12,199	3,050	1,355	0,762	0,488	0,339	0,250	0,191	0,151	0,122	0,054	0,030
	3	27,947	6,887	3,105	1,747	1,118	0,778	0,570	0,437	0,345	0,279	0,124	0,070
	4	64,022	16,006	7,114	4,001	2,561	1,778	1,307	1,000	0,790	0,640	0,285	0,160
	5	146,666	36,667	16,296	9,167	5,867	4,074	2,993	2,291	1,811	1,467	0,652	0,367
	6	335,990	83,997	37,332	20,999	13,440	9,333	6,857	5,250	4,148	3,360	1,493	0,840
I		F _i											
1	53,8298	2,8664	0,7165	0,3187	0,1793	0,1147	0,0797	0,0587	0,0447	0,0355	0,0285	0,0129	0,0070
2	9,7872	1,1939	0,2985	0,1326	0,0746	0,0478	0,0332	0,0245	0,0187	0,0148	0,0119	0,0053	0,0029
3	22,7860	6,3624	1,5907	0,7069	0,3977	0,2545	0,1767	0,1298	0,0995	0,0785	0,0635	0,0282	0,0159
4	5,1064	3,2692	0,8173	0,3633	0,2043	0,1308	0,0908	0,0567	0,0311	0,0403	0,0327	0,0146	0,0082
5	3,1915	4,6808	1,1702	0,5201	0,2926	0,1872	0,1300	0,0955	0,0731	0,0578	0,0468	0,0208	0,0117
6	0,6383	2,1446	0,5362	0,2383	0,1340	0,0858	0,0596	0,0438	0,0335	0,0265	0,0214	0,0095	0,0054
Q, F _i		20,5173	5,1294	2,2799	1,2825	0,8208	0,5700	0,4190	0,3206	0,2534	0,2048	0,0913	0,0511
Q, F _M		169,12	42,28	18,79	10,57	6,77	4,70	3,45	2,64	2,09	1,69	0,75	0,42
I		F _i											
7	2,9787	29,808	7,452	3,312	1,863	1,192	0,828	0,608	0,466	0,368	0,298	0,132	0,074
8	0,8511	16,046	4,011	1,783	1,003	0,642	0,446	0,327	0,251	0,198	0,160	0,071	0,040
9	0,2128	9,668	2,417	1,074	0,604	0,387	0,268	0,197	0,151	0,119	0,097	0,043	0,024
10	0,2128	16,334	4,083	1,815	1,021	0,653	0,454	0,333	0,255	0,201	0,163	0,072	0,041
11	0,2128	10,450	2,613	1,161	0,653	0,418	0,290	0,213	0,163	0,129	0,105	0,046	0,026
13	0,2128	15,223	3,806	1,691	0,951	0,609	0,423	0,311	0,238	0,188	0,152	0,068	0,038
A/701'		97,53	24,33	10,84	6,10	3,90	2,71	1,99	1,52	1,20	0,98	0,43	0,24
n _{vi}		256,65	66,66	29,63	16,67	10,67	7,41	5,44	4,16	3,29	2,67	1,18	0,66

seem more expedient to install them in sites where the technical personnel versed in such matters is easily available (for example, where there already exist industrial plants or other supervised civil engineering works). Consequently, it will not be possible for the seismographic network to be uniform; it will be characterized by the location of the devices in preselected sites.

Then again, installation of the network will have to depend on the position of the epicenters of the earthquakes, which are not uniformly distributed over the entire area of Italian territory; as a matter of fact, the part of Italy most struck by shocks of more than degree V in the period 1893-1965 is about 120,000 km² while it can be assumed that the greater part of earthquakes is concentrated in an area of about 100,000 km² (see the report of Ref. 1 -- seismic maps of Italy for the areas of maximum intensity and for epicenters).

In such an area the dislocation [sic] of about 100 accelerographs installed at a mean distance from one another of about 30 km would make it possible, as we have already seen, to record about 2.5 earthquakes each year with about seven annual recordings.

A better result could be obtained if the 100 devices were placed only in zones where seisms are experienced more frequently, i.e., at a distance from one another that varies as a function of the frequency of the events per unit area. Useful for this purpose may be the figures of Ref. 1, which reports for all of Italy on the frequency of shocks for square areas of 36 km².

A third consideration to be borne in mind for installation /16
of the devices is that the preselected zones be amenable to industrial development or at least be earmarked for constructions of some type.

Indeed, installation of the network should in the first instance serve engineering purposes and should be able both to furnish the information needed for the designing of constructions that are better from the structural point of view and to evaluate safety factors for the people living close to potentially dangerous industrial zones.

A final consideration that cannot be neglected is the economic aspect of the problem. For this purpose even approximate knowledge of the results that can be obtained with a uniform network may be useful for evaluating the time within which an operative network can furnish the engineering data underlying any antiseismic project and hence for deducing correlations between different earthquake parameters.

The investment of capital for the acquisition and installation of a certain number of strong motion accelerographs may therefore entail advantages with respect to the future. In fact, more extreme designing -- that is, with design parameters (acceleration and response spectrum) closer to what is actually experienced at the site -- can actually lead to savings in construction inasmuch as most of the time the values that we are now obliged to assign to design parameters are certainly too prudential.

In making such economic evaluations, finally, it should be borne in mind that additional information can be obtained by combining each accelerograph with a certain number of seismoscopes, which cost considerably less.

In conclusion, the designing of an accelerograph network must take into account the following factors:

/17

- 1) Choice of sites in terms of the availability of maintenance personnel;

2) Location of the instruments in the vicinity of epicentral zones;

3) Installation of the devices near industrial development zones or densely populated zones;

4) Definition of the number of devices on the basis of economic evaluations.

The considerations set forth in the preceding paragraphs can therefore be repeated with due regard for the effective location of the accelerographs, and the number of events that can be picked up, which is associated with the number of obtainable recordings, will be calculated in accordance with the same criteria.

Statistical Analysis of Areas

This analysis is based on study of 470 earthquakes of degree VI or more experienced in Italy from 1893 to 1943 and from 1947 to 1965 for which it was possible to trace out the isoseismals.

The study was carried out on 1 : 600,000 topographic paper, and the areas surrounded by each isoseismal were calculated planimetrically and expressed in cm^2 ($1 \text{ cm}^2 = 36 \text{ km}^2$).

Denoting these areas by A_{ij} , where i denotes the degree of the isoseismal that surrounds the area itself and j denotes the degree of the seism at its epicenter, we will have N sets of n_{ij} dimensions.

Denoting by 1 degree VI, by 2 degree VI-VII ... by 13 degree XII, subscripts i and j will go from 1 to 13 and i will always be less or at most equal to j inasmuch as the investigation was limited to earthquakes of less than degree VI.

The areas taken into consideration are those of whole (and odd) degree of any earthquake of degree j and the epicentral areas of earthquakes of intermediate degree ($i = j$ even).

The lognormal distribution is the one that best fits the N sets of areas A_{ij} .

Setting $x_{ij} = \log A_{ij}$, the x_{ij} sets were divided into constant interval classes $\Delta x = 0.04$ and the number of observations v_{ij} belonging to each class was counted.

In the first column of Table 1 is shown the central value of each class; in the last column, the extreme values of each

X	VI							TABLE I VII							VIII				IX			Class Interval
	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₇	V ₈	V ₉	V ₁₀	V ₁₁	V ₁₂	V ₁₃	V ₁₄	V ₁₅	V ₁₆	V ₁₇	V ₁₈	V ₁₉	V ₂₀		
1.00	9							3	3							1						0.1 - 0.1
1.04	13							4	6							1						0.1 - 0.1
1.08	11							4	6							3	1			1	1	0.1 - 0.1
1.12	6	1	1					6	6							3						0.1 - 0.1
1.16	9							4	4							1				1	1	0.1 - 0.1
1.20	10		1					3	4							1	1					0.1 - 0.1
1.24	13							2	3							1				1	1	0.1 - 0.1
1.28	8	1	1					2	3	1	1					2	2			1		0.1 - 0.1
1.32	7							1	1	1						1				1		0.1 - 0.1
1.36	7							1	2	1						2	1			1		0.1 - 0.1
1.40	7	2	1					1	3							1	1					0.1 - 0.1
1.44	9	1	1					3	5							1				1		0.1 - 0.1
1.48	6	1							2													0.1 - 0.1
1.52	5	1	1					1	5													0.1 - 0.1
1.56	6	1	1					1	1													0.1 - 0.1
1.60	6	1	1					1	1													0.1 - 0.1
1.64	8	1	2					3	3	1							1					0.1 - 0.1
1.68	5		2					1														0.1 - 0.1
1.72	6	2	3		1				2							2		1		1		0.1 - 0.1
1.76	6	2	1					1	4	1									1			0.1 - 0.1
1.80	7	2	1	1				1	4		1					2	3			1		0.1 - 0.1
1.84	7		3					1	2	1						1						0.1 - 0.1
1.88	3	2	1						2		1					1		1				0.1 - 0.1
1.92	9	1	3					1	4	1							1			1		0.1 - 0.1
1.96	7	1	2						2	2						2	1		1			0.1 - 0.1
2.00	8	1	2						3											1	1	0.1 - 0.1
2.04	9	1	3						1				1							1		0.1 - 0.1
2.08	5	2	2	1					3	1								1				0.1 - 0.1
2.12	5	2	2						2	1	1							1				0.1 - 0.1
2.16	3	1	3						3	1										1		0.1 - 0.1
2.20	3		2						1											1		0.1 - 0.1
2.24	7	1	2					1	1	1												0.1 - 0.1
2.28	4		3		1					1	1											0.1 - 0.1
2.32	1	3	2			1			1	1	1					1						0.1 - 0.1
2.36	3	3							3									1				0.1 - 0.1
2.40	2	3	3		1				3	1	1		1									0.1 - 0.1
2.44	3	1	2						2	1	1	1					1					0.1 - 0.1
2.48	4	1	4						2	1	1											0.1 - 0.1
2.52		2	3	3						1												0.1 - 0.1
2.56	1	1	2	1					1	1										1		0.1 - 0.1
2.60		1	2	2	1					2							1					0.1 - 0.1
2.64			2	1	1				1									1	1			0.1 - 0.1
2.68		1	4	3							2									1		0.1 - 0.1
2.72	3		6	1	1							1										0.1 - 0.1
2.76	1	1	4		1								1									0.1 - 0.1
2.80	1		5								1											0.1 - 0.1
2.84		1	1																1			0.1 - 0.1
2.88	2	2	2	1	1		1															0.1 - 0.1
2.92			4					1														0.1 - 0.1
2.96			3	3	1		2			2		1										0.1 - 0.1
3.00			2	1																1		0.1 - 0.1
3.04	1		1	1	1																	0.1 - 0.1
3.08						2			1				1									0.1 - 0.1
3.12									1													0.1 - 0.1
3.16		1																				0.1 - 0.1
3.20																						0.1 - 0.1
3.24																						0.1 - 0.1
3.28																						0.1 - 0.1
3.32																						0.1 - 0.1
3.36																						0.1 - 0.1
3.40																						0.1 - 0.1
3.44																						0.1 - 0.1
3.48																						0.1 - 0.1
3.52																						0.1 - 0.1
3.56																						0.1 - 0.1
3.60																						0.1 - 0.1
3.64																						0.1 - 0.1
3.68																						0.1 - 0.1
3.72																						0.1 - 0.1
3.76																						0.1 - 0.1
3.80																						0.1 - 0.1
3.84																						0.1 - 0.1
3.88																						0.1 - 0.1
3.92																						0.1 - 0.1
3.96																						0.1 - 0.1
4.00																						0.1 - 0.1
4.04																						0.1 - 0.1
4.08																						0.1 - 0.1
4.12																						0.1 - 0.1
4.16																						0.1 - 0.1
4.20																						0.1 - 0.1
4.24																						0.1 - 0.1
4.28																						0.1 - 0.1
4.32																						0.1 - 0.1
4.36																						0.1 - 0.1
4.40																						0.1 - 0.1
4.44																						0.1 - 0.1
4.48																						0.1 - 0.1
4.52																						0.1 - 0.1
4.56																						0.1 - 0.1
4.60																						0.1 - 0.1
4.64																						0.1 - 0.1
4.68																						0.1 - 0.1
4.72																						0.1 - 0.1
4.76																						0.1 - 0.1
4.80																						0.1 - 0.1
4.84																						0.1 - 0.1
4.88																						0.1 - 0.1
4.92																						0.1 - 0.1
4.96																						0.1 - 0.1
5.00																						0.1 - 0.1
5.04					</																	

single class in cm^2 ; and in the central columns, observations v_{ij} .

The subdivision into classes of size 0.04 was kept constant for values of x between -0.08 and 2.44; for classes less than -0.08 difficulties of planimetric surveying led to irregularities in the succession of the classes themselves while for values of x greater than 2.44, no table shows those classes for which v_{ij} turned out to be zero in every set.

Table 2 shows the values of A_{ij} expressed in cm^2 for earthquakes of degree $j = 9, 10, 11, 12, 13$ with $v = 1, 1, 1, 0, 1$.

TABLE 2.

$i =$	1	3	5	7	9	10	11	13
$j = 9$	470	200	146	35	4			
$j = 10$	794	525	253	58	13.4	2.6		
$j = 11$	508	225	115	34.4	4.8		0.7	
$j = 13$	740	290	147	56.5	25		14.4	3.8

Limitedly to the series shown in Table 1 we calculated the /21 mean value \bar{x}_{ij} of the logarithm of areas A_{ij} and of the root-mean-square deviation $s_{x_{ij}}$ as indicated in Table 3. In the last column of Table 3 \bar{R}_{ij} represents the radius in km of the circle equivalent to the area corresponding to mean value \bar{x}_{ij} .

The hypothesis that the areas have a lognormal distribution is confirmed by graphs on lognormal probabilistic paper in which, plotting on the abscissa the values of A_{ij} and on the ordinate the cumulative frequencies F_{ij} that correspond to them in each

TABLE 3.

Degree of Area	Degree of Seism.	n_{ij}	S Σx_v	S^2/n	SS Σx^2_v	SSD $SS - \frac{S^2}{n}$	$s^2_{x_{ij}}$ $\frac{SSD}{n-1}$	$s_{x_{ij}}$	\bar{x}_{ij} $\frac{S}{n}$	\bar{R}_{ij} km
I	VI	253	51,24	10,3776	89,3904	79,0128	0,3135	0,5600	0,2025	4,3
	VI-VII	46	33,64	24,6011	36,7728	12,1717	0,2705	0,5201	0,7313	7,9
	VII	107	107,44	107,8818	138,7136	30,8318	0,2909	0,5393	1,0041	10,8
	VII-VIII	24	33,64	47,1521	51,1792	4,0271	0,1751	0,4184	1,4017	17,0
	VIII	15	21,92	32,0324	35,7472	3,7148	0,2653	0,5151	1,4613	18,2
	VIII-IX	3	5,64	10,6032	10,6448	0,0416	0,0208	0,1443	1,8800	29,5
	IX	14	28,20	56,8029	60,7824	3,9795	0,3061	0,5533	2,0143	34,4
	IX-X	4	9,16	20,9764	21,2144	0,2380	0,0793	0,2817	2,2900	47,3
II	VI-VII	46	-7,00	1,0652	11,6464	10,5812	0,2351	0,4849	1,8478	2,8
	VII	107	24,12	5,4371	43,5696	38,1325	0,3597	0,5998	0,2254	4,4
	VII-VIII	24	18,28	13,9233	18,7664	4,8431	0,2106	0,4589	0,7617	8,1
	VIII	15	15,68	16,3908	20,6592	4,2684	0,3049	0,5522	1,0453	11,3
	VIII-IX	3	4,32	6,2208	6,4736	0,2528	0,1264	0,3555	1,4400	17,8
	IX	14	21,88	34,1953	37,3296	3,1343	0,2411	0,4910	1,5629	20,5
III	IX-X	4	7,36	13,5424	13,7088	0,1664	0,0555	0,2355	1,8400	28,2
	VII-VIII	24	-0,68	0,0193	4,6896	4,6703	0,2031	0,4506	1,9717	3,3
	VIII	15	4,28	1,2212	6,9296	5,7084	0,4077	0,6386	0,2853	4,7
	VIII-IX	3	2,52	2,1168	2,6768	0,5600	0,2800	0,5291	0,8400	8,9
	IX	14	15,20	16,5029	19,1392	2,6363	0,2028	0,4503	1,0857	11,8
	IX-X	4	5,64	7,9524	8,1424	0,1900	0,0633	0,2517	1,4100	17,2
IV	VIII-IX	3	0,52	0,0901	0,8624	0,7723	0,3866	0,6218	0,1733	4,1
	IX	14	4,24	1,2841	5,4144	4,1303	0,3177	0,5637	0,3029	4,8
	IX-X	4	3,60	3,2400	3,5040	0,2640	0,0880	0,2967	0,9000	9,5
V	IX-X	4	-0,40	0,0400	0,7584	0,7184	0,2395	0,4893	1,9000	3,0

single series of data, we get as coordinate points $(A_{ij}; F_{ij})$ -- at least for values of $F(x)$ between 0.15 and 0.85; in conformity with our hypothesis, they lie approximately on the straight line $F(x)$, no point falling outside of the fiduciary limits.

Figure 1, for example, shows the graph on lognormal probabilistic paper of the series with subscripts $i = j = 1$ for $n_{11} = 253$ data. The statistical series of recorded values of x_{ij} , frequency density $f(x)$ and cumulative frequency curve $F(x)$ that can be deduced from it, yield a synthetic picture of what happened in the period of years under consideration; in any case it would be arbitrary to draw from this picture inferences about what is going to happen in the future or, more precisely, they are indistinguishable from the probabilities that correspond to them. In fact, the values of $f_{ij}(x)$ and $F_{ij}(x)$ derived for an assigned value of x_{ij} of the available series of n_{ij} data only represent a more or less rough estimate of probability density $p(x_{ij})$ or cumulative probability $P(x_{ij})$ belonging to x_{ij} , and we should expect them to vary when we go to another period of years different from that to which the observations refer.

Analogously, the values of \bar{x}_{ij} and $s_{x_{ij}}$ calculated for each series represent only the estimates that the existing empirical series furnishes of mean value $\xi(x_{ij})$ and of root-mean-square deviation $\sigma(x_{ij})$, which characterize the probability distribution of areas A_{ij} . If by subscript t is denoted the position of the area with respect to the epicentral area such that by $t = 1$ is indicated the epicentral area of earthquakes of whole degree $i = 1, 3, 5, 7$; by $t = 2$, the second area, i.e., the area for which $i = j - 1$; by $t = 3$, the third area, i.e., the area for which $i = j - 2$; and so forth until $t = 6$ for the areas for which $i = j - 5$; if, finally, by $t = 0$ is denoted the epicentral area of earthquakes of intermediate degree $j = 2, 4, 6, 8$,

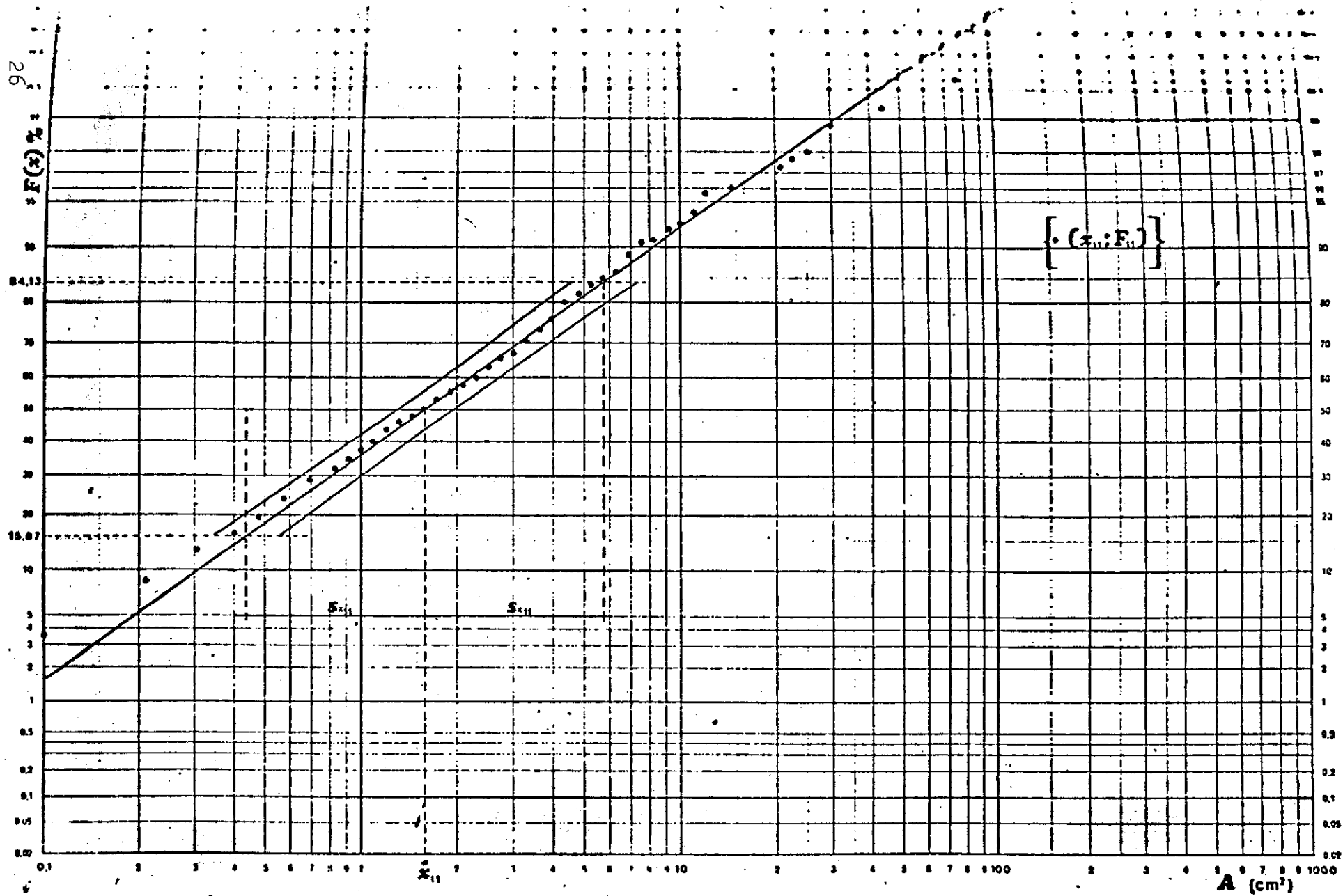


Fig. 1.

the series shown in Table 3 can be gathered into two groups in which the area under consideration has the same position with respect to the epicentral area. The group of epicentral areas for $j = 2, 4, 6$, and 8 will be considered separately.

In each group mean values \bar{x}_{ij} turn out to be quite constant and -- at least for those series for which the data are more abundant -- root-mean-square deviations $s_{x_{ij}}$ have values that do not differ very much from one another.

In practice, it would seem that the distribution of the areas of each group is the same for all the series that compose the group itself; in other words, the distribution of the areas about a mean value does not depend on the degree of intensity but on the position that the area under consideration occupies with respect to the epicentral area.

This hypothesis was checked by means of an analysis of variance, whose results are shown in Table 4.

In accordance with the hypothesis set forth above, in Table 4 the new weighted means of the seven groups are denoted by \bar{x}_t with t going from 0 to 6; by N_t is denoted the number of the series that compose the group; by s_{x_t} , the root-mean-square deviation of each group calculated by means of the following equation:

$$s_{x_t}^2 = \frac{\sum_{i=1}^{N_t} \sum_{j=1}^{n_{ij}} (x_{ij} - \bar{x}_{ij})^2}{\sum_{i=1}^{N_t} n_{ij}} \quad (1)$$

based on the deviations between the single elements x_{ij} and their arithmetic mean \bar{x}_{ij} that can be found within each series; by v^2 , is denoted the value calculated by means of the following equation

$$v^2 = s_{\bar{x}_t}^2 / s_{x_t}^2 \quad (2)$$

where

$$s_{\bar{x}_t}^2 = \frac{\sum_1^{N_t} n_{ij} (\bar{x}_{ij} - \bar{x}_t)^2}{N_t - 1} \quad (3)$$

and, finally, the value $v^2(f_1; f_2)_{0.95}$ is given, which is the 95% fractile of $P\{v^2(f_1; f_2)\} = 0.95$, where by $f_1 = N_t - 1$ and $f_2 = \sum_1^{N_t} n_{ij} - N_t$ are denoted the degrees of freedom of statistical function v^2 . This function depends only on the two values of the degrees of freedom, and its probability distribution was studied and tabulated for different combinations of the values assumed by f_1 and f_2 , so that the values of $v^2(f_1; f_2)$ corresponding to assigned values of probability $P\{v^2\}$ are known.

Inasmuch as $v_{0.95}^2$ yields, according to convention, a measure of the maximum value that might still be expected for v^2 in case the N statistical series should be derived from populations distributed according to the same probability law and characterized by one and the same value of the mean and of the root-mean-square deviation and all the $v^2 < v_{0.95}^2$, we can conclude that our hypothesis is fully acceptable.

Making the most of all the information furnished by the data, we assume as estimate of median ξ_t and of root-mean-square deviation σ_t of the probability function $P_t(x)$ of group t , mean value \bar{x}_t and root-mean-square deviation s_t calculated by means of the following two equations:

$$\bar{x}_t = \frac{\sum_1^{N_t} \sum_1^{M_t} x_{ij} n_{ij}}{\sum_1^{N_t} n_{ij}} \quad (4)$$

TABLE 4.

Group	Series		n_{ij}	\bar{x}_{ij}	N_i	\bar{x}_i	$s_{x_i}^2$	$s_{\bar{x}_i}^2$	f_1	f_2	v^2	$v_{0.95}^2$
	i	j										
					$\frac{\sum_{ij} n_{ij}}{N_i}$	$\frac{\sum_{ij} n_{ij} \bar{x}_{ij}}{\sum_{ij} n_{ij}}$	$\frac{\sum_{ij} n_{ij} (\bar{x}_{ij} - \bar{x}_i)^2}{\sum_{ij} n_{ij} - N_i}$	$\frac{\sum_{ij} n_{ij} (\bar{x}_{ij} - \bar{x}_i)^2}{N_i - 1}$	$N_i - 1$	$\sum_{ij} n_{ij} - N_i$	$s_{\bar{x}_i}^2 / s_{x_i}^2$	
$t=0$	2	2	46	0.1522								
	4	4	24	0.0283								
	6	6	3	0.1733								
	8	8	4	0.1000								
					4	77	-0.0982	0.22934521	0.15751665	3	73	0.69 2.74
$t=1$	1	1	253	0.2025								
	3	3	107	0.2254								
	5	5	15	0.2853								
	7	7	14	0.3029								
					4	389	0.2156	0.32982857	0.07068576	3	385	0.21 2.63
$t=2$	1	2	46	0.7313								
	3	4	24	0.7617								
	5	6	3	0.8400								
	7	8	4	0.9000								
					4	77	0.7538	0.24436712	0.04419147	3	73	0.18 2.74
$t=3$	1	3	107	1.0041								
	3	5	15	1.0453								
	5	7	14	1.0857								
					3	136	1.0171	0.28373308	0.04794752	2	132	0.17 3.07
$t=4$	1	4	24	1.4017								
	3	6	3	1.4400								
	5	8	4	1.4100								
					3	31	1.4065	0.15963929	0.00198436	2	28	0.01 3.34
$t=5$	1	5	15	1.4613								
	3	7	14	1.5629								
					2	29	1.5103	0.25367037	0.07474964	1	27	0.29 4.21
$t=6$	1	6	3	1.8800								
	3	8	4	1.8400								
					2	7	1.8571	0.04160000	0.00274287	1	5	0.07 6.61

$$s_t^2 = \frac{\sum_{i=1}^{N_t} \sum_{j=1}^{M_{ij}} (x_{ij} - \bar{x}_t)^2}{\sum_{i=1}^{N_t} n_{ij} - 1} \quad (5)$$

The effective values of ξ_t and σ_t may differ more or less sharply from \bar{x}_t and s_t , but by means of analysis of variance it is possible to define the fiduciary limits within which they should fall.

In fact, if we indicate by the following equation

$$s_x^2 = \left[\sum_{i=1}^{N_t} n_{ij} \right] (\bar{x}_t - \xi_t)^2 \quad (6)$$

the ratio between the two different estimates S_x^2 and S_t^2 of σ^2 , it is distributed like $v^2(f_1; f_2)$ with $f_1 = 1$ and $f_2 = \sum_{i=1}^{N_t} n_{ij} - 1$ degrees of freedom and, having derived value $v^2(f_1; f_2)_{0.95}$, we can therefore assume, in fiduciary terms, the following:

$$v^2(f_1; f_2) \leq v^2(f_1; f_2)_{0.95} \quad (7)$$

Or even, knowing that in the particular case in which $f_1 = 1$ the distribution of $v^2(f_1; f_2)$ refers to that of t^2 with f_2 degrees of freedom and

$$t^2 = \left\{ \frac{\bar{x}_t - \xi_t}{s_t} \right\}^2 \quad (8)$$

the well known function from which $P\{t\}$ is known, we can

assume that t should have an absolute value less than $|t_{0.025}|$ (or $|t_{0.975}|$, inasmuch as the distribution of t is symmetric with respect to zero), in which, as usual, by $|t_{0.025}|$ ($|t_{0.975}|$) is denoted the value which, in the distribution of t for

$f = f_2 = \sum_{j=1}^{N_1} n_{1j} - 1$, is assumed by the fractile of $P\{t\} = 0.025$
 $(P\{t\} = 0.975)$

Denoting value $|t_{0.025}|$ by the letter K, we get as fiduciary limits of ξ the following values: /25

$$\xi' = \bar{x}_1 + K s_1 / \sqrt{\sum_{j=1}^{N_1} n_{1j}} \quad (9) \quad \text{and} \quad \xi'' = \bar{x}_1 - K s_1 / \sqrt{\sum_{j=1}^{N_1} n_{1j}} \quad (10)$$

With an analogous criterion, finally, we can calculate the fiduciary limits of σ_1^2 and σ_0^2 of σ^2 .

In fact, since analysis of variance shows that S_t^2/σ^2 is distributed like χ^2/f with $f = f_2 = \sum_{j=1}^{N_1} n_{1j} - 1$ degrees of freedom, by denoting by $(\chi^2/f)_{0.025}$ and $(\chi^2/f)_{0.975}$ the values of χ^2/f that for $f = f_2$ degrees of freedom turn out to be fractiles of $P\{\chi^2/f\} = 0.025$ and $P\{\chi^2/f\} = 0.975$, we can define σ_1^2 and σ_0^2 by means of the following relations:

$$\sigma_1^2 = s_1^2 / (\chi^2/f)_{0.025} \quad (11) \quad \sigma_0^2 = s_1^2 / (\chi^2/f)_{0.975} \quad (12)$$

with $f = f_2 = \sum_{j=1}^{N_1} n_{1j} - 1$ degrees of freedom.

Probability functions $P\{x\}$ and $\bar{P}\{x\}$ defined by the values \bar{x}_t and s_t represent each series of the t -th sample.

The two samples $t = 1$ and $t = 0$, while both representing the epicentral areas, have different mean values; this difference may perhaps be due to uncertainty in defining the intermediate degree.

Table 5 shows the values of \bar{x}_t and s_t that we found and the fiduciary limits within which must fall ξ and σ and radius \bar{R}_t of the circle equivalent to area \bar{A}_t of mean value \bar{x}_t . Figure 2 shows in cartesian coordinates the values of \bar{R}_t derived from \bar{x}_t , ξ' and ξ'' for each value of t .

Plotting in cartesian coordinates (Fig. 3) points $(t; \bar{x}_t)$ for t going from 1 to 6, we notice that the points are fairly aligned and that the straight line interpolated between them has the value 0.36 as its angular coefficient. Nor does point $(0; \bar{x}_0)$ stray from the interpolation line, as though it represented the next smallest area after the epicentral area of the whole degrees. /26

These considerations led us to formulate the hypothesis that the probability distributions of the t groups of areas have the same root-mean-square deviation and mean values, which vary in accordance with the following law:

$$\bar{x}_t = 0,24 + 0,36(t - 1) \quad (13)$$

This hypothesis was checked with analysis of variance, whose results are shown in Table 6.

In this table, v^2 denotes the calculated value of the ratio of the two estimates S_t and S_{x_t} of σ ; $v^2(f_1; f_2)_{0.95}$ denotes the values of v^2 with $f_1 = N_t - 1$ and $f_2 = \sum_{i=1}^{N_t} n_{ij} - N_t$ degrees of freedom corresponding to probability $P\{v^2(f_1; f_2) = 0,95$

The values of s_{x_t} and s'_t can be derived from Eq. (1) and the following equation

$$s'_t = \frac{\sum_{i=1}^{N_t} n_{ij} (\bar{x}_{ij} - \bar{x}_t)}{N_t - 1} \quad (14)$$

TABLE 5.

t	0	1	2	3	4	5	6
N_t	4	4	4	3	3	2	2
$\sum_{j=1}^{N_t} n_{ij}$	77	389	77	136	31	29	7
f	76	388	76	135	30	28	6
k	1.992	1.968	1.992	1.977	2.042	2.048	2.447
\bar{x}_t	-0.0982	0.2156	0.7538	1.0171	1.4065	1.5103	1.8571
ξ'_t	0.0098	0.2727	0.8642	1.1068	1.5481	1.6994	2.0304
ξ''_t	-0.2062	0.1585	0.6434	0.9274	1.2649	1.3212	1.6838
$(\chi^2_t)_{0.025}$	0.7077	0.8641	0.7077	0.7757	0.5597	0.5467	0.2062
$(\chi^2_t)_{0.975}$	1.3421	1.1457	1.3421	1.2523	1.5660	1.5879	2.4082
s_t	0.4759	0.5726	0.4863	0.5294	0.3862	0.4973	0.1874
σ'_t	0.5657	0.6160	0.5780	0.6011	0.5162	0.6725	0.4127
σ''_t	0.4108	0.5350	0.4197	0.4731	0.3086	0.3947	0.1208
\bar{R}_t	3.02	4.34	8.06	10.92	17.10	19.27	28.72

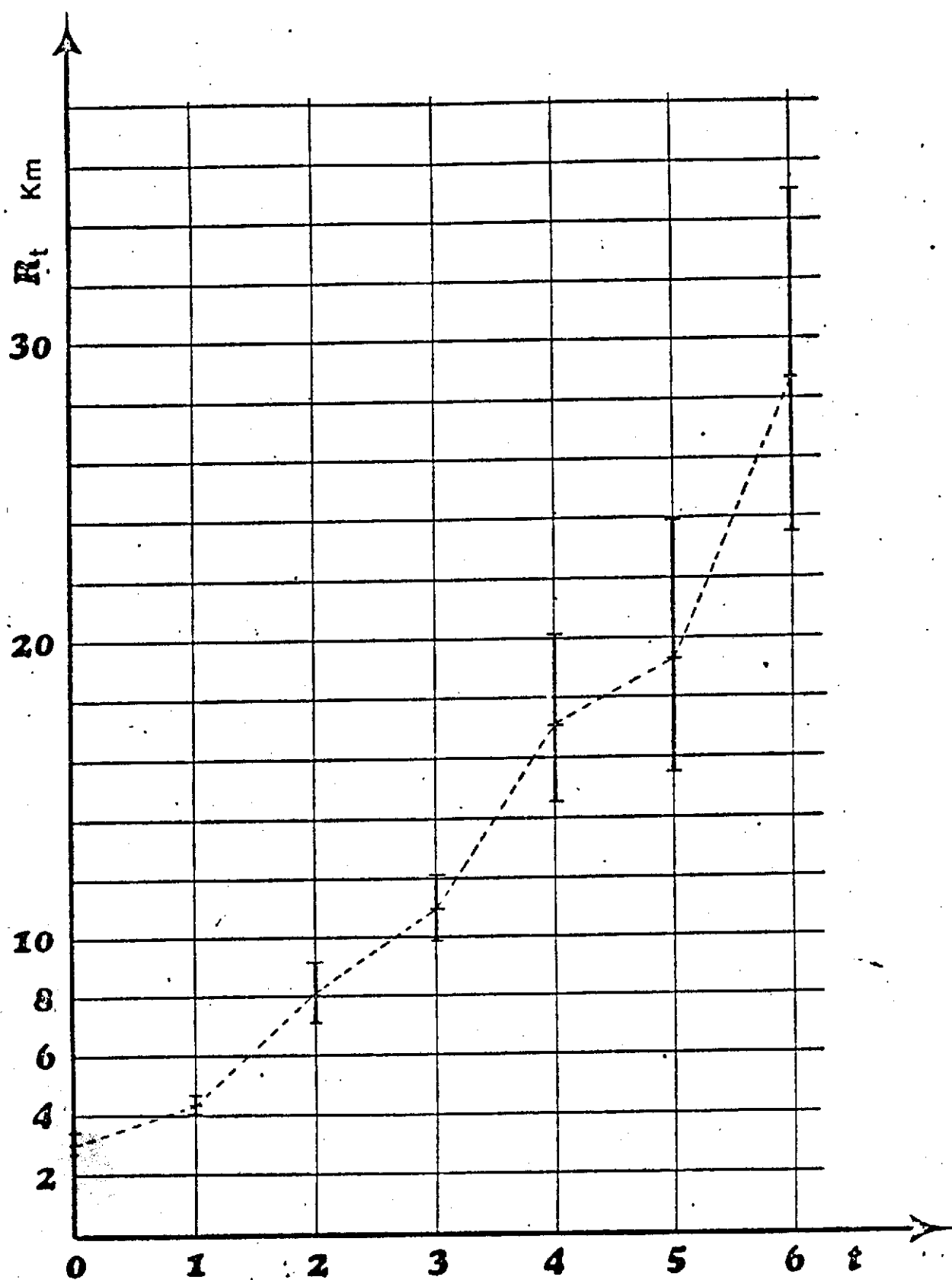


Fig. 2.

t	Series		n _{ij}	\bar{x}_{ij}	\bar{x}_i	TABLE 6.		f_1	f_2	u^2	$u_{0.95}^2$
	i	j				$s_{x_i}^2$	s_i^2				
0	2	2	46	-0.1522							
	4	4	24	-0.0283							
	6	6	3	0.1733							
	8	8	4	-0.1000							
					-0.12	0.22934521	0.16972756	3	73	0.74	2.74
1	1	1	253	0.2025							
	3	3	107	0.2254							
	5	5	15	0.2853							
	7	7	14	0.3029							
					0.24	0.32982857	0.15492015	3	385	0.47	2.63
2	1	2	46	0.7313							
	3	4	24	0.7617							
	5	6	3	0.8400							
	7	8	4	0.9000							
					0.60	0.24436712	0.65111703	3	73	2.66	2.74
3	1	3	107	1.0041							
	3	5	15	1.0453							
	5	7	14	1.0857							
					0.96	0.28373308	0.26922144	2	133	0.95	3.07
4	1	4	24	1.4017							
	3	6	3	1.4400							
	5	8	4	1.4100							
					1.32	0.15963929	0.11789868	2	28	0.74	3.34
5	1	5	15	1.4613							
	3	7	14	1.5629							
					1.68	0.25367037	0.90941909	1	27	3.59	4.21
6	1	6	3	1.8800							
	3	8	4	1.8400							
					2.04	0.04160000	0.23680000	1	5	5.69	6.61

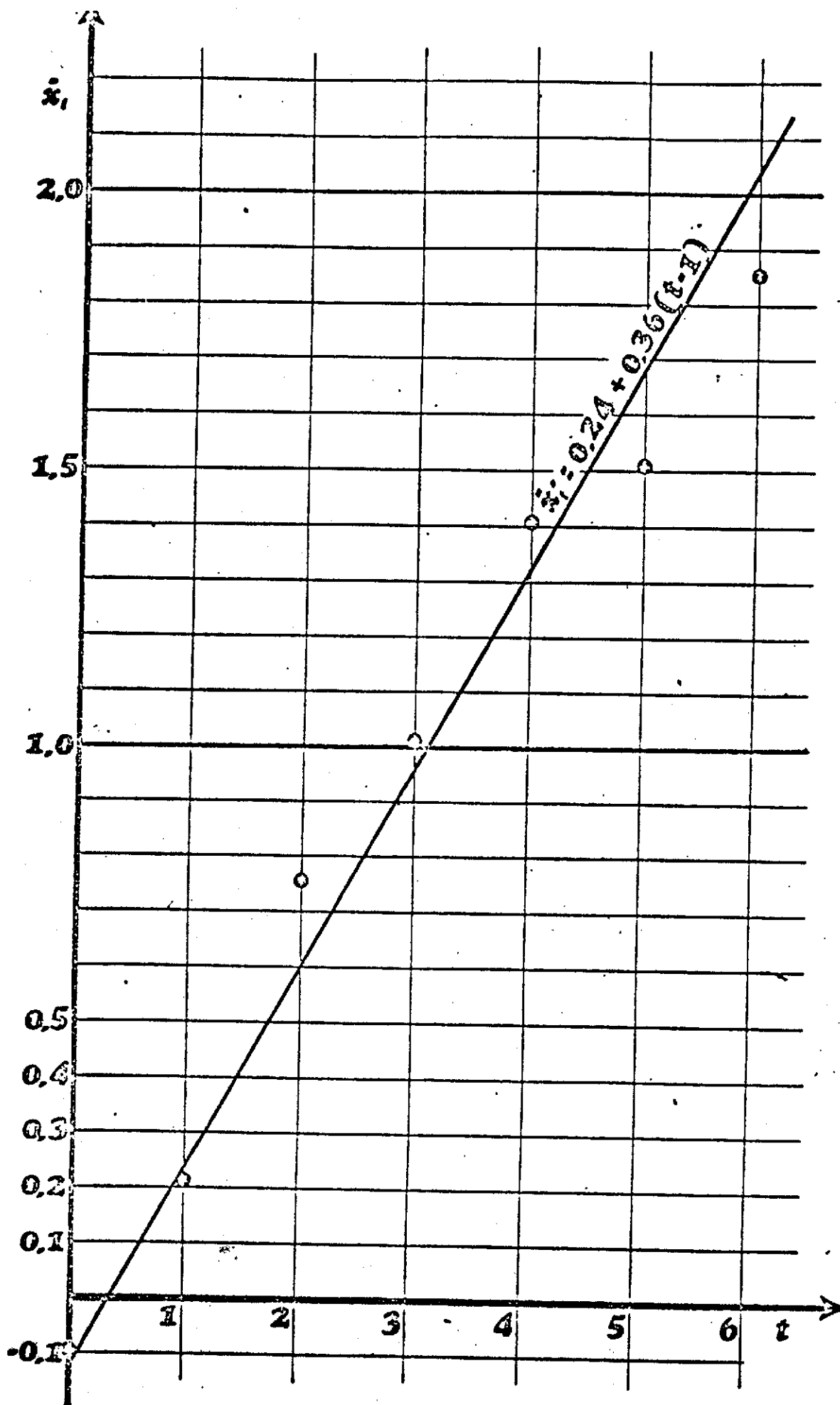


Fig. 3.

Inasmuch as all of the values of v^2 are less than the corresponding $u_{0.95}^2$, we can conclude that our hypothesis is fully acceptable.

As root-mean-square deviation of each series, we adopt the value calculated with the following equation

$$S^2 = \frac{\sum_{i=0}^6 \sum_{j=1}^{N_i} n_{ij} (x_{ij} - \bar{x}_i)^2}{\sum_{i=0}^6 \sum_{j=1}^{N_i} n_{ij} - 1} = 0.2856 \quad (15)$$

and for the mean value of each series of the t -th group, the value calculated by means of Eq. (13).

In general, we will have:

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$$P_t \left\{ x \right\} = \frac{1}{\sqrt{2\pi}S} \int_{-\infty}^x e^{-\frac{(x-0.24-0.36(t-1))^2}{2S^2}} dx \quad (16)$$

$$P_{t+2} \left\{ x + 0.36 z \right\} = P_t \left\{ z \right\} \quad (17)$$

Our hypothesis does not, however, permit us to say that ratio A_{t+1}/A_t is statistically distributed around a mean value; but, in order to facilitate calculations, it is possible to use a single probability curve and to pass from one series to the other by means of Eqs. (16) and (17).

Finally, inasmuch as $A_{ij} = \pi R_{ij}^2$, where R_{ij} denotes the radius of the circle equivalent to area A_{ij} , and in passing to the logarithms, the root-mean-square deviation referred to the logarithms of radii S_R is half of that referred to the logarithms of the areas, we give the probability functions as a function of radius R expressed in km.

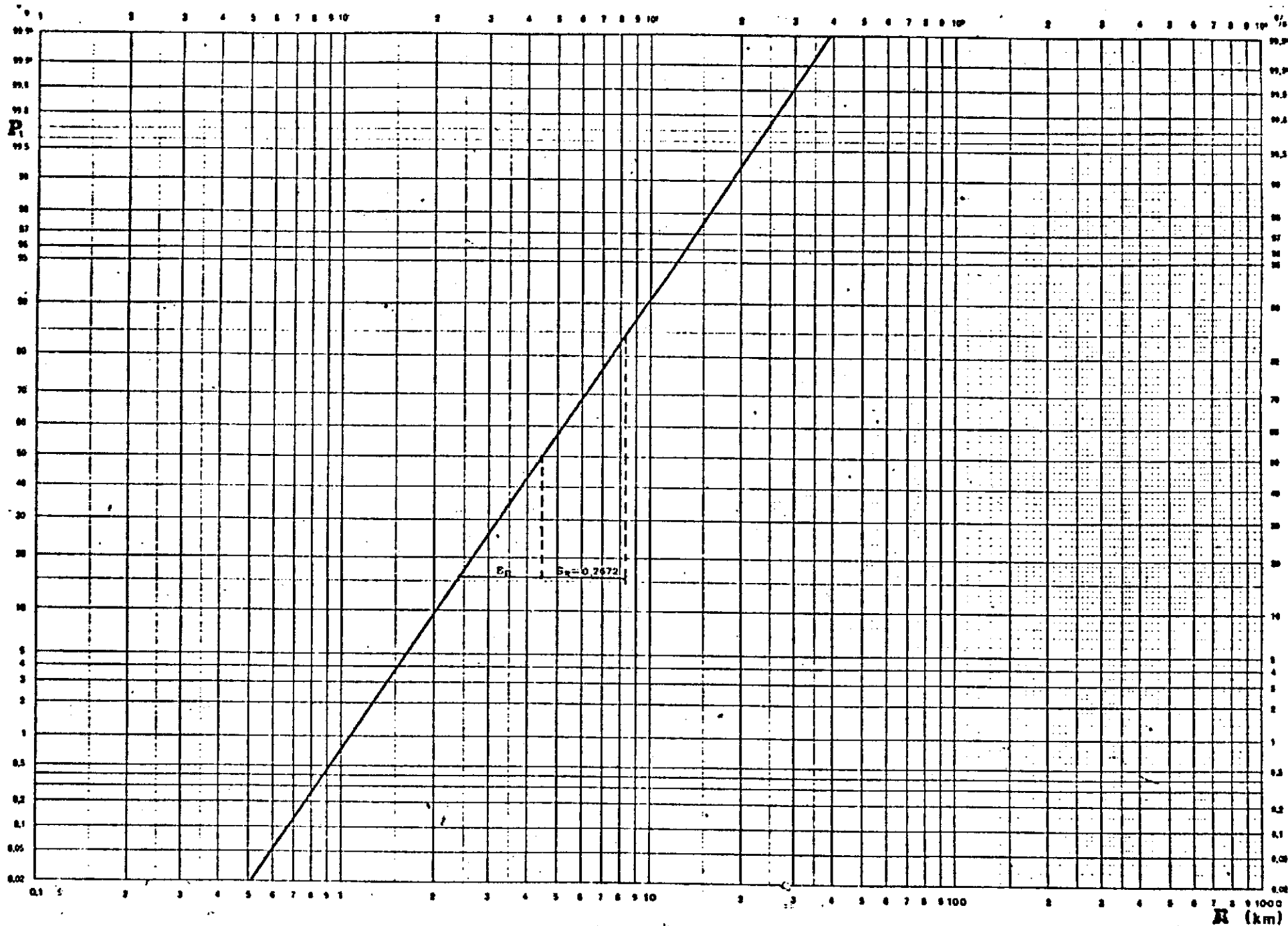


Fig. 4.

Since $\bar{x}'_1 = 0.24$; $\bar{x}'_{t+1} - \bar{x}'_t = 0.36$ and $s = 0.5345$, we get $\bar{R}'_1 = 4.46$ km; $\bar{R}'_{t+1} - \bar{R}'_t = \sqrt{2.291} = 1.5136$ and $s_R = 0.2672$.

In general, we can write:

$$P_t \left\{ R \right\} = \frac{1}{\sqrt{2\pi} s_R} \int_0^R e^{-\frac{(\log R - 0.64933 - 0.18(t-1))^2}{2s_R^2}} \frac{dR}{R} \quad (18)$$

$$P_{t+1} \left\{ \log R + 0.18 \quad z \right\} = P_t \left\{ \log R \right\} \quad (19)$$

Figure 4 shows on lognormal probabilistic paper the cumulative probability curve $P_t \left\{ R \right\}$ for equal and lesser values of R .

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